

# QFT In Physical Systems: Condensed Matter to Life Sciences

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## Abstract

A panoramic view is given of the applications of QFT to diverse areas of physics, beginning with the concepts of QED Vacuum as well as of electron self-energy and renormalization. The related concepts of the degenerate vacuum, spontaneous symmetry breaking (SSB) and Nambu-Goldstone bosons, together with their applications to condensed matter physics and quasi particles are outlined. The possibilities of application to life sciences are also examined. Finally the alternative method of Path integrals for introducing QFT is briefly indicated.

## 1 Introductory background

Quantum Field Theory (QFT) has had a long history of evolution since the time Dirac postulated his famous ‘Sea’ of negative energy states, to the present day when efforts are seriously on for the application of its methodology all the way to Brain Dynamics [1]. It all started with the single particle non-relativistic quantum mechanics (QM) formulation by Heisenberg and Schroedinger who did not have to face any consistency problem, until Dirac –and Klein-Gordon before him– introduced relativity into the picture. Noting the failure of the (second order ) Klein-Gordon equation to produce a positive definite probability density, Dirac attempted a first order equation in matrix form (did he have the analogy to the Maxwell equations in mind

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?), and thus avoided the problem of negative probability, but only to find a second bottleneck-the existence of negative energy states ! He eventually overcame this problem through his novel theory of the positron : a hole in the sea of negative energy states, caused by the absorption of enough external energy ( $> 2mc^2$ ) by a (negative energy) electron to get lifted to a positive energy state, thus creating a particle-hole pair. With the discovery (by C D Anderson) of a cosmic ray particle with precisely the properties of the "hole", he romped home despite Pauli's determined attack against his interpretation [2]. Yet Pauli must have gauged the true significance of Dirac's result, as was to be evidenced by his own successful quantization of the Klein-Gordon equation [3] through a reinterpretation of the non-positive definite probability density as an average charge density of both types of charges, positive and negative. Some years later, Dyson [4] offered a clear perspective which covered both the situations (Dirac sea of spin-half fermions as well as the Pauli-Weisskopf average charge density of scalar bosons.) : For a consistent theory which includes both Relativity and QM, one must take an infinite number of particles for mathematical consistency-in other words a field description with infinite d.o.f.'s . In this sense QFT is the right generalization from the QM for a single particle ! Of course QFT had routinely been a part of quantum theory since the early thirties , but its true significance became clear after this conceptual input by Dyson [4]; see also Mitra [5].

This paper is intended to serve as an elementary introduction to the application of QFT methodology to 'soft' condensed matter physics as a preliminary to applications to life-science (with a proper role for the environment)[1]. To that end we collect some essential preliminaries in the next two sections on the methodology of QED ( the first non-trivial example of QFT), including definition of the vacuum state (Sect 2), and ideas of self-energy and renormalization (Sect 3). The concept of the degenerate vacuum is introduced next in Sect 4, including those of Spontaneous Symmetry Breaking (SSB) and long range Nambu-Goldstone Bosons. The applications of QFT to standard Condensed Matter Physics (CMP), with special reference to phonons and magnons, are outlined in Sect 5, while some additional techniques relating to life sciences (which must now include the role of environment as well as unidirectional flow of time ) are taken up in Sect 6. Finally in Sect 7 the method of Path integrals is outlined as an alternative means to access QFT, especially for its role in incorporating the effect of environment for an open system.

## 2 QFT language: Vacuum State

The QFT generalization of single particle QM is now routinely considered as the basic framework for applications of quantum theory to diverse physical problems. The specific language employed for the purpose is almost exclusively that of a harmonic oscillator characterized by the appearance of both the canonical variables  $\{x, p\}$  in a quadratic form in the energy operator, which in certain units is expressible as  $H = (p^2 + \omega^2 x^2)/2$ . And because of the symmetrical appearance of  $\{x, p\}$ , it is usually convenient to express the energy operator in terms of their standard complex combinations  $\{a, a^+\}$  which in dimensionless units read

$$a = (-i\omega x\hbar + p)/\sqrt{2\omega\hbar}; \quad a^+ = (+i\omega x\hbar + p)/\sqrt{2\omega\hbar} \quad (1)$$

$a$  and  $a^+$  satisfy the commutation relations  $[a, a^+] = 1$ , while other commutators vanish. The energy operator,  $H = \hbar\omega(a^+a + 1/2)$  incorporates the essential dynamics in terms of the number operator  $a^+a$  whose integer eigenvalues  $n$  are called occupation numbers and the corresponding ‘states’ (wave functions) are called the eigenstates  $|n\rangle$  of  $a^+a$ . By virtue of the equalities  $a|n\rangle = \sqrt{n}|n-1\rangle$  and  $a^+|n\rangle = \sqrt{n+1}|n+1\rangle$ ,  $a$  and  $a^+$  are called destruction / creation operators respectively, since they reduce / increase the occupation number in a given state  $|n\rangle$  by one unit each. Hence by successive applications of the  $a$  operators, the eigenstates  $|n\rangle$  are all expressible in terms of the ground state (‘vacuum’)  $|0\rangle$  as proportional to  $a^{+n}|0\rangle$ , noting that  $a|0\rangle = 0$ . In this language the successive eigenvalues of the energy operator are  $E_n = \hbar\omega(n + 1/2)$ . The QFT generalization for an infinite collection of harmonic oscillators  $a_k, a_k^+$  indexed by  $k$  (another integer), is now immediate: Since  $[a_k, a_{k'}^+] = \delta_{k,k'}$  incorporates all the commutation relations, a general eigenstate of this infinite collection of harmonic oscillators  $|\{n_k\}\rangle$  is compactly expressible as  $\prod_k a_k^{+n_k}|0\rangle$ , where  $|0\rangle$  now represents the “master” ground state, or Vacuum, of the infinite collection of harmonic oscillators. [There is no problem of ‘ordering’ of these creation operators since they all commute].

This “master” ground state – or simply Vacuum state – is a central theme around which the entire concept / methodology of QFT devolves. [Rather paradoxically, the vacuum concept owes its origin to Dirac’s Hole Theory not only for fermions (for which it was intended) but also for bosons through an obvious extension of the ideas]. And in those days the vacuum was considered as a unique ground state precisely of the type described in the foregoing.

The earliest application of the QFT formalism was in the area of quantum electrodynamics -QED for short-the theory of interaction of charged particles (electrons, free or bound) with the bosonic electromagnetic (e.m.) field. This of course required a ‘vector’ formulation of the harmonic oscillator field, a feature already present in the energy density (  $\mathbf{E}^2/2 + \mathbf{H}^2/2$  ) of the Maxwell field. Its Fourier analysis, apart from routine technical details, yields creation and destruction operators of the e.m. quanta together with a polarization index  $\lambda$ , viz.,  $a_{k\lambda}$ ,  $a_{k\lambda}^+$ , which stand respectively for the absorption / emission of a radiation quantum of momentum  $k$  and polarization  $\lambda$ , by an electron.

### 3 QED: Self-Energy and Renormalization

QED has had an intense two-decade old history (1930-50) of vicissitudes with regard to experiment, wherein its spectacular successes for lowest order processes like Compton scattering (scattering of e.m. radiation quanta by free electrons) and Bremsstrahlung (scattering of electrons by the coulomb field of an atomic core, with the simultaneous emission of a quantum of radiation) have co-existed with total failures for processes involving higher order e.m. effects, namely, prediction of infinities which are totally incompatible with experiment. Typically, a higher order e.m. process consists in the emission and re-absorption of a photon, grafted on an otherwise lowest order process (such as Compton scattering above); this may be described as a second order correction to the latter. Now since energy-momentum conservation does not put any restriction on the magnitude of the (virtual) photon momentum emitted and reabsorbed by the electron, the amplitude for the full process must be a sum of all sub-processes corresponding to all possible momenta from zero to infinity. And this sum is what gives rise to a (linearly) divergent result on integration over  $k$ . This kind of divergence-called the electron’s *self – energy* by virtue of the capacity of the electron to emit and reabsorb a photon– is typical of the problems faced with QED processes whenever higher order corrections to a given lowest order process are attempted. Perhaps these had best be ignored, as some physicists had argued then, were it not for an unexpected development in the mid Forties bearing on experiment: The microwave techniques developed during World War II facilitated more accurate measurements of the energy levels of atoms like hydrogen and helium than had been possible hitherto. In particular it was found by Lamb and Retherford that the  $2p_{1/2}$  and  $2s_{1/2}$  levels of hy-

drogen differ from the original Dirac prediction of identical values (hitherto in excellent agreement with experiment) by about 1040 megacycles ! This led to intense discussions in which the infinities inherent in the second order radiative corrections to atomic processes of the type described above figured prominently. Eventually at the Solvay Conference of 1947 it was agreed that the idea of Renormalization should be seriously considered for interpreting the electron's e.m. self-energy ( defined above) as indistinguishable from its total observed mass. Now since the value of the electron's self energy varies according to its environment, it became logical—for purposes of lumping this infinity with the mechanical mass of the electron— to identify some universal value which should be independent of its environment; a natural candidate being the self-energy of a *free* electron ! Therefore the prescription became clear : From the self energy of an electron bound in, say, a hydrogen atom, subtract a universal part independent of its environment, viz., that of a free electron. The resultant quantity should hopefully be finite and hence measurable. The programme, although attractive, had to be extremely elaborate so as to make the result truly independent of any observer. This in turn needed a fully covariant formulation of QED, one in which three stalwarts (Tomanaga, Schwinger, Feynman) participated, catalysed by a fourth one (Dyson), and the entire programme took several years for the essential features of the new theory to be fully implemented In the meantime a provisional estimate by Hans Bethe [6] indicated excellent agreement with experiment. The rest is history.

## 4 Degenerate Vacuum : SSB and NG-Bosons

So far we have considered the interaction of the pure electron and e.m. fields ( QED) whose vacuum is unique (non-degenerate) i.e., a well-defined state of *lowest* energy. In this background the concepts of Self -energy, and Renormalization discussed above must be regarded as second (and higher) order e.m. effects. Now the uniqueness of the QED vacuum is the result of certain standard symmetries-translational, rotational, Lorentz-as well as the non-trivial symmetry of gauge invariance which accounts for the conservation of charge. Let us next consider a more general QFT (in the background of other fields) when one or more of such symmetries are no longer satisfied. In such a generalized QFT scenario, the Vacuum is no longer unique, rather *degenerate*, characterized by the phenomenon of Spontaneous Sym-

metry Breaking (SSB) to the accompaniment of certain *massless* (as well as spinless !) quanta called Nambu-Goldstone (NG) bosons (which necessarily have an infinite range). The historical developments in this sector have been aptly summarised by Weinberg [7] in terms of the role of symmetries in the development of the physics of the Twentieth Century, from spacetime to internal symmetries. Thus ‘spontaneously broken’ symmetries are those that are not realized as symmetry transformations of the physical states of the theory, and are always associated with a degeneracy of vacuum states. In particular, for spontaneously broken continuous symmetries there is a theorem that, for each broken symmetry, the spectrum of physical particles must contain one particle of zero mass and spin. [The theory must not dictate which member is distinct, only that one of the members is]. Such particles—called Nambu-Goldstone bosons— which correspond to the spontaneously broken internal symmetry generators, and are characterized by their quantum numbers— were first encountered by Nambu [8] in the context of BCS superconductivity, as well as by Goldstone [9] in a specific model; after this two general proofs of their existence were given by Goldstone, Salam and Weinberg [10] within the framework of QFT.

These spinless bosons transform nonlinearly (shift) under the action of the generators of SSB, and can thus be excited out of the ( now asymmetric) vacuum by these generators. Thus, they can be thought of as (collective) excitations of the field in the broken symmetry directions in group space— and are massless if the spontaneously broken symmetry is not also broken explicitly. If, on the other hand, the symmetry is not exact, i.e., if in addition to being spontaneously broken, it is also explicitly broken, then the Nambu-Goldstone bosons are not massless, although they typically remain relatively light; they are then called pseudo-NG bosons. Condensed matter physics abounds in pseudo NG bosons, or simply pseudoparticles. Some typical examples are :

- i) In fluids, the phonon is longitudinal ; it is the NG boson of the spontaneously broken ‘Galilean’ symmetry (wrt the transformation  $x' = x - vt$ ). On the other hand in solids, the NG bosons (phonons) have both longitudinal and transverse modes, corresponding to spontaneously broken Galilean, translational, and rotational symmetries; but there is no simple one-to-one correspondence between such modes and the broken symmetries.
- ii) In magnets, the original rotational symmetry (when no external magnetic field is present) is spontaneously broken such that the magnetization points into a specific direction. The NG bosons are then called magnons, i.e., quanta

of spin waves in which the local magnetization direction oscillates. In addition if an external magnetic field is present, the rotational symmetry is also explicitly broken, in which case the magnons also acquire a small mass.

iii) As a third example— this time from the field of elementary particles— the pions are the pseudo-NG bosons that result from the spontaneous breakdown of the chiral symmetries of QCD as a result of quark condensation (a typical strong interaction effect). These symmetries are further explicitly broken by the masses of the quarks, so that the pions are not entirely massless, but their mass is significantly smaller than typical hadron masses.

## 5 SSB in condensed matter physics

Perhaps the most extensive use of the concept of a degenerate vacuum in QFT has been in the domain of condensed matter physics (CMP) where the facilities of experimentation are more readily available than in the (more expensive) domain of particle physics. This makes it possible to test the more intricate features of the theory of vacuum structure in much greater detail than would be feasible in any other comparative field. Indeed in the CMP sector, there is a virtual goldmine of the corresponding NG bosons – phonons, magnons, plasmons, excitons, polarons, polaritons, to name a few–, giving rise to a spate of discoveries like superconductivity and superfluidity; Bose-Einstein condensation and Josephson effect, all of which have been experimentally confirmed. For purposes of illustration however we choose only two typical NG-bosons – phonons and magnons –. and briefly indicate the steps for their dynamical working as a consequence of SSB in the corresponding vacuum (including the Bogoliubov transformation). In the following it will be instructive to quote selectively from Kittel’s book [11] to illustrate the ideas involved.

### 5.1 Phonons as NG bosons

Our first example of SSB in QFT – the phonon – is the name for the field quanta of elastic excitations in a crystal. In terms of the creation and annihilation operators (  $a_k$  ,  $a_k^+$  ) , SSB generally causes their mixing so as to violate the standard conservation of the corresponding QFT number operators  $a_k^+ a_k$ . Such mixing in turn leads to certain linear combinations ( $\alpha_k$  and  $\alpha_k^+$ ) emerging out of the original  $a_k$ ,  $a_k^+$  pairs ( known as the Bogoliubov trans-

formation), thus defining new (conserved) number operators  $\alpha_k^+ \alpha_k$ , whose quanta are now the NG bosons. However, in the simplest case of elastic lattice (discrete) vibrations from crystals one already obtains a diagonalized Hamiltonian (even before a Bogoliubov transformation!) in the standard form [12] :

$$H = \sum \omega_k (n_k + 1/2); \quad \omega_k = \sqrt{2(1 - \cos k)}; \quad n_k = a_k^+ a_k$$

The number of phonons ( $n_k$ ), in certain units, in the state  $k$  is a non-negative integer. As a less trivial example, phonons can arise in a condensed Bose gas of weakly interacting particles through a similar treatment, but now there also appear terms of the type  $a_k, a_{-k}$  and  $a_k^+, a_{-k}^+$  in the Hamiltonian, and one now needs for its diagonalization a Bogoliubov transformation [12] :

$$\alpha_k = u_k a_k - v_k a_{-k}^+; \quad \alpha_k^+ = u_k a_k^+ - v_k a_{-k}$$

where  $(u_k, v_k)$  are real functions of  $k$  (momentum). The corresponding dispersion relation (behaviour of  $\omega_k$  as a function of  $k$ ) is a straight line for small  $k$  (acoustic mode), but has a pronounced dip at larger  $k$  ; this dip has a built-in mechanism for understanding the phenomenon of superfluidity in a condensed Bose - Einstein gas in liquid helium 4 near absolute zero of temperature (see [12] for details).

## 5.2 Magnons as NG bosons

A second example of QFT techniques concerns the rotational dynamics of spin [13] which encompasses the all-important concepts of spin magnetic moment and the associated magnetic field (internal and external). With spin d.o.f.'s playing the main role, the low-lying energy states of such systems coupled by exchange interactions are wave-like, giving rise to 'spin waves' . The quantization of their energy then gives rise to quanta called magnons. The basic dynamics of such spin systems with nearest neighbour interactions in the presence of an external magnetic field  $H_0$  (along the z-axis) is given by the Heisenberg interaction Hamiltonian applicable to a ferromagnet :

$$H = -J \sum \mathbf{S}_j \cdot \mathbf{S}_{j+\delta} - g\mu_B H_0 \sum S_{jz}$$

where  $J > 0$  is the exchange integral ;  $\delta$  defines the range of nearest neighbour interactions;  $g\mu_B$  is the magnetic moment; and  $\mathbf{S}_j$  is the spin angular



momentum of the atom at location  $j$ . The (x,y,z) components of each  $\mathbf{S}_j$  are connected by the (j-independent) condition [13]

$$\mathbf{S}_j \cdot \mathbf{S}_j = S(S+1)$$

The crucial step needed to bring in the standard creation and destruction operators of QFT is now the Holstein-Primakoff transformation [13] :

$$\begin{aligned} S_j^+ &= S_{jx} + iS_{jy} = \sqrt{2S} \sqrt{(1 - a_j^+ a_j / 2S)} a_j \\ S_j^- &= S_{jx} - iS_{jy} = \sqrt{2S} \sqrt{(1 - a_j^+ a_j / 2S)} a_j \end{aligned} \quad (2)$$

The number operators  $a_j^+ a_j$ 's in turn are connected to the  $S_{jz}$ 's by the relation  $S_{jz} = S - a_j^+ a_j$ . The transformation from the atomic ( $a_j$ ) to the magnon ( $b_k$ ) variables is now achieved by the usual Fourier transformation connecting the two sets [13], whence the latter can be shown to satisfy the standard commutation relations  $[b_k, b_{k'}^+] = \delta_{kk'}$ . The total spin operator  $\mathcal{S}_\dagger$  is now expressed by

$$\mathcal{S}_\dagger = NS - \sum b_k^+ b_k$$

where  $N$  is the number of atoms each of spin  $S$ . With the help of these results, the magnon Hamiltonian becomes a function of the  $b_k, b_k^+$  variables which can be expanded perturbatively in powers of the scalar  $\mathbf{k}b\dot{f}\delta^2$ . This quantity which reduces to  $(ka)^2$  for several standard lattices with a lattice constant  $a$ , is usually small, thus justifying a perturbation treatment. The diagonalization of the Hamiltonian – without a Bogoliubov transformation – then yields a dispersion relation of the form [13] :

$$\omega_k = g\mu_B H_0 + 2JS(ka)^2$$

This result agrees with the corresponding data on magnon dispersion relations in magnetite as determined from inelastic neutron scattering [13]. A similar treatment goes through for an antiferromagnet as well, but now the effective d.o.f.'s get doubled since the spin structure of the crystal get divided into two sets of interpenetrating sublattices  $a$  and  $b$  with the property that all nearest neighbours of an atom on  $a$  lie on  $b$  and vice versa [13]. As a result we now have two sets of QFT operators  $a_j, a_j^+$  and  $b_j, b_j^+$  which therefore require a Bogoliubov transformation for the diagonalization of the corresponding Hamiltonian. The dispersion relation for  $\omega_k$  is now more complicated, but shows a linear behaviour for a simple cubic lattice for small  $ka$ .

## 6 QFT Application to life sciences

We are now in a position to extend the QFT formulation to soft CMP for possible applications to biological systems. There are now two additional features to be taken into account: i) effect of environment; and ii) unidirectional flow of time (amounting to a dissipative dynamics). To implement these features we borrow a model of brain dynamics due to Freeman and Vitiello [14], which is an extension– to dissipative dynamics– of a many-body model proposed by Umezawa et al [15, 16] which states[14]:

“in any material in CMP any particular information is carried by certain ordered patterns maintained by certain long range correlations mediated by massless quanta. It looked to me (Umezawa) that this is the only way to memorize some information; memory is a particular pattern of order supported by long range correlations.”

The long range correlations dynamically generated by SSB via NG-bosons (zero mass modes spanning the whole system) are thus crucial to the ‘memory’ mechanism envisaged by Umezawa. And the coherent condensation of NG bosons in the ground state (vacuum) of the system gives it the appearance of an ‘ordered’ state. A measure of the degree of ordering (or coherence) is provided by the vacuum density of the NG bosons. This measure is also called the order parameter which is a classical field “labelling” the observed order pattern [17]. The recall of ordered information occurs under a stimulus able to excite the density wave quanta (DWQ) out of the ground state (vacuum). Such a stimulus is called “similar” to the one responsible for the memory recording.

Unfortunately this many-body model by itself cannot explain the observed co-existence of amplitude modulation (AM) patterns and their irreversible time evolution [17]. For, any subsequent stimulus ‘cancels’ the previous recorded memory by renewing the SSB process, thus overprinting the new memory over the previous one—a memory capacity problem [17]. According to del Giudice et al [17], as well as Vitiello [14], the many-body model [15] did not consider an *open* system in permanent interaction with the environment, thus missing the key ingredient of *Dissipation*.

Now Dissipation is a relatively new idea whose insertion into the QFT framework involves borrowing from both the picture of the ‘Dirac sea’ (playing the role of environment), as well as Feynman’s novel concept of “Zig-

zagging in time” [18], a feat which Vitiello et al [19] performed by *doubling* the system’s d.o.f.’s as follows

## 6.1 Environ as time-reversed image of system

External stimulus leads to SSB which in turn causes dynamical generation of density wave quanta  $A_k$ . A crucial step is now to double the number of d.o.f.’s to  $\{A_k, \tilde{A}_k\}$ , where  $\tilde{A}_k$  is the time reversed mirror image. [ The analogy is to the ‘method of images’ in electricity to ensure ‘equipotentiality’ of the surface in front of a charge, the surface now playing the role of ‘environment’!]. Next define the corresponding number operators  $N_{Ak} = A_k^\dagger A_k$ , together with a similar expression for  $N_{\tilde{A}k}$ . Then the energy flux balance is

$$E_0 = E_{syst} - E_{env} = \sum \hbar \Omega_k (N_{Ak} - N_{\tilde{A}k}) = 0 \quad (3)$$

Note the similarity of the operators  $\{A_k, \tilde{A}_k\}$  to the electron and ‘hole’ d.o.f.’s in the Dirac sea. To pursue the analogy further, in CMP, the Fermi sea has  $E < E_F$  for hole states, and  $E > E_F$  for electron states. Again in nuclear physics, the ground state corresponds to the vacuum, while the excited states consist of particle-hole pairs.

But the environment is a more complex system in which *dissipation* is a key feature manifesting through some sort of ‘damping’ in the energy operator arising from the two sets  $A_k$  and  $\tilde{A}_k$  of quasi massless operators, while their commutation relations retain their standard structures. The damping effect now shows up through the part  $H_I$  of the total energy operator  $H = H_0 + H_I$ , while the ‘real part’  $H_0$  is symmetrical between the two sets:

$$\begin{aligned} H_0 &= \sum_k \hbar \Omega_k [A_k^\dagger A_k - \tilde{A}_k^\dagger \tilde{A}_k] \\ H_I &= i \sum_k \hbar \Gamma_k [A_k^\dagger \tilde{A}_k^\dagger - A_k \tilde{A}_k] \end{aligned} \quad (4)$$

The two parameters  $\Omega_k$  and  $\Gamma_k$  can now be recognized as the real and imaginary parts of the frequency variable relating to the ‘k-mode’. Note that the ‘damping’ parameter  $\Gamma_k$  does not exist in standard QFT, while the energy flux balance

$$\sum_k \hbar \Omega_k (N_{Ak} - N_{\tilde{A}k}) = 0$$

where the  $N$ 's are non-negative integers, is satisfied in the usual manner. The 'vacuum' is now made up of two sets of zero occupation numbers, so that states can be built up by a straightforward doubling of the techniques of Sect 2. Further, in view of the commutation relation,

$$[H_0, H_I] = 0$$

the combinations  $(N_{Ak} - N_{\bar{A}k})$  are constants of motion for any  $k$ , so that  $H_0$  remains bounded from below if bounded at some initial time  $t_0$ . For other details see ref [19].

The new theory produces several agreements with observations which the many-body theory of Umezawa et al [15] could not explain. These include the QFT dissipative dynamics; (quasi-)non-interfering degenerate vacua; AM pattern textures (phase transitions among them); AM patterns sequencing; and huge memory capacity.

## 7 An alternative formulation : Path Integral Method

For the sake of completeness, we shall outline an alternative formulation for the introduction of QFT language in physics, viz., Feynman's Path Integral method [20] (a book with no references !) for a novel yet holistic understanding of quantum mechanics (without using the language of operators). In its simplest form, it is a sum over all possible *classical* trajectories, each with a definite *phase* which is proportional to  $\exp i \int dt L(x, t)/\hbar$ , where  $\int dt L(x, t)$  is the classical "action" (time-integral over the Lagrangian) over a given path, so that the total amplitude is a sum over all such contributions [20]:

$$\sum_{all\ paths\ from\ a\ to\ b} const \exp i \int dt L(x, t)/\hbar \quad (5)$$

where the constant is subject to (later) adjustment to give the correct normalization. Before proceeding further we pause to make some preliminary comments on the significance of this fundamental expression, especially its correspondence with the precise quantum mechanical amplitude it is supposed to represent. Historically, this form for the phase factor was first conjectured by Dirac in 1931, but not pursued further, until Feynman two

decades later [21] gave a more precise interpretation and developed the ideas systematically to show its equivalence to conventional quantum mechanics, together with the rules of evaluation of the integrals associated with the “master expression” (5) [20]. To describe its structure in a nutshell, it represents the *complete amplitude* for the propagation of a particle from an initial space-time point  $a$  to a final space-time point  $b$ , (also called the Feynman propagator). In the conventional quantum description, this quantity is the result of a long-drawn calculation (using standard techniques) to arrive at this elaborate structure. In this alternative description, on the other hand, this “master expression” is the *starting point* from which it is possible to arrive at, say, the Schroedinger equation by working backwards [20], and in so doing establishing a consistency with the latter.

The principal strategy for the evaluation of (5) is first to define in a systematic way the *measure* of the multiple integrations involved therein, (all of which are amenable to successive gaussian integrations), and then carry out the integrations through a suitable ‘discretization’ procedure, together with a final limiting process—all in a highly pragmatic (no rigour !) manner. An important advantage of this “Path Integral” method whose input is the classical action  $S = \int dt L(x, t)$  is that all quantum mechanical phases are already built-in within entirely *classical* premises. And any quantum mechanical amplitude of physical interest can be derived from this form merely with the help of suitably-defined functional derivatives. In the general case when the quantum system consists of a mixture of states (not a pure one), the corresponding amplitude is called a *density matrix*.

A second advantage of the Path Integral method is that it is trivially adaptable to the techniques of statistical mechanics merely with the replacement of  $i \int_0^t dt L(x, t)/\hbar$  by  $-\int_0^{\beta\hbar} d\tau H(x, \tau)$ , where the new variable  $\tau = -it$  is purely imaginary (called the Matsubara time) and has a maximum value  $\beta\hbar$  which is related to the temperature  $T$  by  $kT = 1/\beta$ . [ Note that this has also necessitated a simultaneous replacement of the Lagrangian  $L$  by the Hamiltonian  $H$  ]. It is remarkable that with such a simple change from a real to imaginary ‘time’ the scope of the Path Integral technique has got vastly extended to be applicable to the entire field of statistical mechanics, with almost no extra charge. A more familiar name for the ‘density matrix’ for the statistical system is the “Partition function”, which goes to show that latter is calculable in terms of the former, simply by extending the scope of the time variable to imaginary values (Matsubara time).

## 7.1 Application to “open” Systems

Still another advantage of the Path Integral method is that with little extra input, it can be extended to calculate the behavior of a system of interest, even when it is coupled to other external quantum systems, in terms of its own variables only. The necessary formalism is due to Feynman and Vernon [22] who have shown that the effect of the external systems in such a formalism can always be included in a general class of functionals – termed influence functionals– of the coordinates of the given system only. To that end it is first necessary to extend the action functional for the path integral to include the variables of the external system (environment) as well. Such variables represent, e.g., the effect of classical forces, linear dissipative systems at finite temperatures, as well as combinations of them. The Feynman-Vernon method is particularly applicable to linear systems composed of combinations of harmonic oscillators, loss being introduced by continuous distributions of oscillators. The resultant quantity may be called the generalized density matrix for the combined system, which, after integration over the external variables is called the reduced density matrix, whose principal components are the “Influence functionals”. The form of the latter is particularly simple for linear dissipative systems, namely, they have the same form as obtainable in terms of their classical response functions.

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